FRACTURE TOUGHNESS AS A FUNCTION OF THE MODE MIXITY USING NON-CONVENTIONAL TECHNIQUES

Filipe J.P. Chaves
IDMEC- Pólo FEUP
L.F.M. da Silva
M.F.S.F. de Moura
DEMc- FEUP
D. Dillard
ESM – Virginia Tech
motivation

fracture modes

fracture tests [mode I, mode II and mixed-mode I+II]

fracture tests [envelope]

conventional techniques [data reduction scheme]

non-conventional techniques [mixed-mode I+II]

conclusions

future work

acknowledgments
Bonded joints in service are usually subjected to mixed-mode conditions due to geometric and loading complexities.

Consequently, the fracture characterization of bonded joints under mixed-mode loading is a fundamental task.

There are some conventional tests proposed in the literature concerning this subject, as is the case of the asymmetric double cantilever beam (ADCB), the single leg bending (SLB) and the cracked lap shear (CLS).

Nevertheless, these tests are limited in which concerns the variation of the mode-mixity, which means that different tests are necessary to cover the fracture envelope in the $G_I-G_{II}$ space.

This work consists on the analysis of the different mixed mode tests already in use, allowing to design an optimized test protocol to obtain the fracture envelope for an adhesive, using a Double Cantilever Beam (DCB) specimen and a Compliance Based Beam Method (CBBM) for the data reduction scheme.
fracture modes for adhesive joints

Figure 1. Fracture modes.

Mode I – opening mode (a tensile stress normal to the plane of the crack);

Mode II – Sliding mode (a shear stress acting parallel to the plane of the crack and perpendicular to the crack front);

Mode III – tearing mode (a shear stress acting parallel to the plane of the crack and parallel to the crack front)
Mode I release rate energy $G_I$ is well known and well characterized.

**DCB – Double Cantilever Beam** & **TDCB – Tapered Double Cantilever Beam**

**Figure 2.** DCB specimen and test.

**Figure 3.** TDCB specimen and test.
Mode II release rate energy $G_{\text{IIC}}$.

<table>
<thead>
<tr>
<th>ENF – End Notch Flexure</th>
<th>4ENF - 4 Points End Notch</th>
<th>ELS – End Load Split</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="ENF Specimen and Test" /></td>
<td><img src="image2.png" alt="4ENF Scheme" /></td>
<td><img src="image3.png" alt="ELS Scheme" /></td>
</tr>
</tbody>
</table>

Figure 3. ENF specimen and test.

Figure 4. 4 ENF scheme

Figure 5. ELS scheme

fracture tests [mode II]
Mixed-Mode I + II release rates energies $G_T = G_{IC} + G_{IIC}$.

Figure 6. Conventional test schemes for mixed-mode I + II.

fracture tests [mixed-mode I+II]
Specimens [DCB, ATDCB, SLB, ENF]

**Bondline thickness = 0.2 mm**

<table>
<thead>
<tr>
<th>Table 1. Adhesive shear properties using the thick adherend shear test method ISO 11003-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Shear modulus ( G ) (MPa)</td>
</tr>
<tr>
<td>Shear yield strength ( \tau_{ya} ) (MPa)</td>
</tr>
<tr>
<td>Shear strength ( \tau_f ) (MPa)</td>
</tr>
<tr>
<td>Shear failure strain ( \gamma_f ) (%)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2. Steel adherend properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
</tr>
<tr>
<td>Young modulus, ( E ) [Gpa]</td>
</tr>
<tr>
<td>Yield strength, ( \sigma_y ) [MPa]</td>
</tr>
<tr>
<td>Shear strength, ( \sigma_y ) [MPa]</td>
</tr>
<tr>
<td>Strain, ( \varepsilon_f ) [%]</td>
</tr>
</tbody>
</table>

Figure 7. DCB, ATDCB, SLB and ENF specimen geometries.
Table 3. Fracture toughness obtained with the conventional testing methods (average and standard deviation).

<table>
<thead>
<tr>
<th>Test type</th>
<th>$G_{IC}$ (N/mm)</th>
<th>$G_{IIc}$ (N/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCB⁺</td>
<td>0.44 ± 0.05</td>
<td>-</td>
</tr>
<tr>
<td>ENF</td>
<td>-</td>
<td>2.1 ± 0.21</td>
</tr>
<tr>
<td>SLB⁺</td>
<td>0.34 ± 0.06</td>
<td>0.32 ± 0.06</td>
</tr>
<tr>
<td>ADCB⁺</td>
<td>0.41 ± 0.04</td>
<td>0.004 ± 0.0005</td>
</tr>
<tr>
<td>ATDCB⁺</td>
<td>0.32 ± 0.04</td>
<td>0.07 ± 0.006</td>
</tr>
</tbody>
</table>

+ Value corresponding to CBBM
* For these tests, the values indicated correspond to $G_I$ and $G_{II}$ and not $G_{IC}$ and $G_{IIc}$.

Figure 8. Fracture envelope for conventional tests.
Data reduction schemes

**Compliance Calibration Method** (CCM) is based on the Irwin-Kies theory (Trantina 1972, Kanninen and Popelar 1985)

**Direct Beam Theory** (DBT), based on elementary beam theory (Ding 1999)

**Corrected Beam Theory** (CBT) (Robinson and Das 2004, Wang and Williams 1992)

The Compliance Based Beam Method (CBBM) was recently developed by de Moura et al. (2008, 2009) and is based on the crack equivalent concept.
The Dual Actuator Load Frame (DAL) test is based on a DCB specimen loaded asymmetrically by means of two independent hydraulic actuators. Different combinations of applied displacement rates provide different levels of mode ratios, thus allowing an easy definition of the fracture envelope in the $G_I$ versus $G_{II}$ space.

Classical data reduction schemes based on compliance calibration and beam theories require crack length monitoring during its growth, which in addition to FPZ ahead of the crack tip can be considered important limitations.

**non-conventional techniques** [mixed-mode I+II]
Using Timoshenko beam theory, the strain energy of the specimen due to bending and including shear effects is:

\[
U = \int_0^a \frac{M_R^2}{2EI_R} \, dx + \int_0^a \frac{M_L^2}{2EI_L} \, dx + \int_0^a \frac{M_T^2}{2EI} \, dx + \int_0^a \int_{-h/2}^{h/2} \frac{\tau_R^2}{2G} B \, dy \, dx + \int_0^a \int_{-h/2}^{h/2} \frac{\tau_L^2}{2G} B \, dy \, dx + \int_a^L \int_{-h}^{h} \frac{\tau_T^2}{2G} B \, dy \, dx
\]

(1)

\( M \) is the bending moment

subscripts \( R \) and \( L \) stand for right and left adherends

\( T \) refers to the total bonded beam (of thickness \( 2h \))

\( E \) is the longitudinal modulus

\( G \) is the shear modulus

\( B \) is the specimen and bond width

\( I \) is the second moment of area of the indicated section

For adherends with same thickness, considered in this analysis, \( I = 8I_R = 8I_L \)

Figure 12. Schematic representation of loading in the DAL test.
The shear stresses induced by bending are given by:

\[
\tau = \frac{3}{2} \frac{V}{Bh} \left(1 - \frac{y^2}{c^2}\right)
\]

(2)

c - beam half-thickness
V - transverse load → on each arm for \(0 \leq x \leq a\), and on total bonded beam for \(a \leq x \leq L\)

From Castigliano’s theorem

\[
\delta = \partial U / \partial P
\]

\[
\left\{ \begin{array}{l}
P \text{ is the applied load} \\
\delta \text{ is the resulting displacement at the same point}
\end{array} \right.
\]

the displacements of the specimen arms can be written as

\[
\delta_L = \frac{\left(7a^3 + L^3\right)F_L}{2Bh^3E} + \frac{(L^3 - a^3)F_R}{2Bh^3E} + \frac{3L\left(F_L + F_R + a(F_L - F_R)\right)}{5BhG}
\]

\[
\delta_R = \frac{\left(7a^3 + L^3\right)F_R}{2Bh^3E} + \frac{(L^3 - a^3)F_L}{2Bh^3E} + \frac{3L\left(F_L + F_R + a(F_R - F_L)\right)}{5BhG}
\]

(3)
The DAL test can be viewed as a combination of the DCB and ELS tests.

![Figure 13. Schematic representation of loading in the DAL test.](image)

**Figure 13.** Schematic representation of loading in the DAL test.

\[
P_I = \frac{F_R - F_L}{2}
\]

\[
P_{II} = F_R + F_L
\]

\[
\delta_I = \delta_R - \delta_L
\]

\[
\delta_{II} = \frac{\delta_R + \delta_L}{2}
\]

**non-conventional techniques** [mixed-mode I+II]
Combining equations (3-5), the pure mode compliances become

\[ C_I = \frac{\delta_I}{P_I} = \frac{8a^3}{Bh^3E} + \frac{12a}{5BhG} \]  

and

\[ C_{II} = \frac{\delta_{II}}{P_{II}} = \frac{3a^3 + L^3}{2Bh^3E} + \frac{3L}{5BhG} \]  

However, stress concentrations, root rotation effects, the presence of the adhesive, load frame flexibility, and the existence of a non-negligible fracture process zone ahead of crack tip during propagation are not included in these equations.

To overcome these drawbacks, equivalent crack lengths can be calculated from the current compliances \( C_I \) and \( C_{II} \) (eq. 6 and 7)

\[ \alpha a_{cl}^3 + \beta a_{cl} + \gamma = 0 \]

\[ \alpha = \frac{8}{Bh^3E}; \quad \beta = \frac{12}{5BhG}; \quad \gamma = -C_I \]

\[ A = \left( -108\gamma + 12\sqrt{3\left( \frac{4\beta^3 + 27\gamma^2\alpha}{\alpha} \right)\alpha^2} \right)^{1/3} \]

\[ a_{el} = \frac{1}{6\alpha} A - \frac{2\beta}{A} \]

\[ a_{eII} = \left[ \left( C_{II} - \frac{3L}{5BhG} \right) \frac{2Bh^3E}{3} - \frac{L^3}{3} \right]^{1/3} \]

non-conventional techniques [mixed-mode I+II]
The strain energy release rate components can be determined using the Irwin-Kies equation:

\[
G = \frac{P^2}{2B} \frac{dC}{da}
\]

(10)

\[
C_1 = \frac{\delta_I}{P_I} = \frac{8a^3}{Bh^3E} + \frac{12a}{5BhG}
\]

combined with equation 6

\[
C_\text{II} = \frac{\delta_{\text{II}}}{P_{\text{II}}} = \frac{3a^3 + \bar{L}^3}{2Bh^3E} + \frac{3L}{5BhG}
\]

combined with equation 7

\[
G_1 = \frac{6P_I^2}{B^2h} \left( \frac{2a_{\text{cl}}^2}{h^2E} + \frac{1}{5G} \right)
\]

(11)

\[
G_{\text{II}} = \frac{9P_{\text{II}}^2 a_{\text{II}}^2}{4B^2 h^3 E}
\]

(12)

The method only requires the data given in the load-displacement (P-δ) curves of the two specimen arms registered during the experimental test.

Accounts for the Fracture Process Zone (FPZ) effects, since it is based on current specimen compliance which is influenced by the presence of the FPZ.
Numerical analysis including a cohesive damage model was carried out to verify the performance of the test and the adequacy of the proposed data reduction scheme.

Table 4. Elastic and cohesive properties.

<table>
<thead>
<tr>
<th>Elastic properties (Steel)</th>
<th>Cohesive properties (Adhesive)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (GPa)</td>
<td>G (MPa)</td>
</tr>
<tr>
<td>210</td>
<td>80.77</td>
</tr>
</tbody>
</table>

The specimen was modelled with 7680 plane strain 8-node quadrilateral elements and 480 6-node interface elements with null thickness placed at the mid-plane of the bonded specimen.

*non-conventional techniques [mixed-mode I+II]*
non-conventional techniques

Figure 15. ABAQUS simulation mixed-mode (left) mode I (right)

Figure 16. The linear softening law for pure and mixed-mode cohesive damage model.

### Figure 16

\[ \frac{\sigma_i}{\sigma_{u,I}} + \frac{\sigma_{II}}{\sigma_{u,II}} = 1 \]  \hspace{1cm} (13)

\[ \left( \frac{G_i}{G_{lc}} \right) + \left( \frac{G_{II}}{G_{IIc}} \right) = 1 \]  \hspace{1cm} (14)

\[ \frac{G_i}{G_{lc}} \] and \[ \frac{G_{II}}{G_{IIc}} \] are the linear energetic criterion to deal with damage growth.
It is useful to define the displacement ratio $\lambda = \frac{\delta_l}{\delta_R}$

$$\begin{align*}
\lambda &= -1 & \text{pure mode I} \\
\lambda &= 1 & \text{pure mode II}
\end{align*}$$

Figure 17. Normalized $R$-curves for the pure modes loading: a) Mode I; b) Mode II.
six different cases were considered in the range \(-0.9 \leq \lambda \leq -0.1\) in this case mode I loading clearly predominate

nine combinations were analysed for \(0.1 \leq \lambda \leq 0.9\) a large range of mode ratios is covered

Table 5. Imposed displacements for each simulation

<table>
<thead>
<tr>
<th>Simul. #</th>
<th>imposed displacem.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>beam 1</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>14</td>
<td>10</td>
</tr>
<tr>
<td>15</td>
<td>10</td>
</tr>
</tbody>
</table>
for $\lambda = 0.7$, the $R$-curves vary as a function of crack length variation of mode-mixity as the crack grows.

Figure 18. $R$-curves for $\lambda = 0.7$ (both curves were plotted as function of $a_{el}$ for better comparison).
the condition of self-similar crack growth is not satisfied

the proportion of mode II component increases as the crack propagates, being sensibly constant close to the end of the test

\[ G_T = G_I + G_{II} \]

Figure 19. Evolution of mode mixity during propagation for \( \lambda = 0.7 \).

FPZ length \( l_{FPZ} \) varies during crack growth as a consequence of the mode-mixity variation

\( l_{FPZ} \) is approximately constant near to the end of the test

is in agreement with the approximately constant mode-mixity observed in figure 19.

Figure 20. FPZ length during crack growth.
the curves were cut at the beginning of the inflexion caused by the referred effects

non-conventional techniques [mixed-mode I+II]
Figure 22. Plot of the $G_I$ versus $G_{II}$ strain energy components for $-0.9 \leq \lambda \leq -0.1$.

- Non-conventional techniques [mixed-mode I+II]

- Mode I predominant loading conditions

- Combinations $-0.9 \leq \lambda \leq -0.7$, are nearly pure mode I loading conditions

- Linear energetic criterion
combinations using the positive values of $\lambda$ induce quite a large range of mode ratios during crack propagation.

excellent reproduction of the inputted linear criterion in the vicinity of pure modes, presenting a slight difference where mixed-mode loading prevail explained by the non self-similar crack growth, which is more pronounced in these cases.

Figure 23. Plot of the $G_I$ versus $G_{II}$ strain energies for $0.1 \leq \lambda \leq 0.9$. 

non-conventional techniques [mixed-mode I+II]
practically the entire fracture envelope can be obtained using only two combinations ($\lambda = 0.1$ and $\lambda = 0.75$)

**Important advantage of the DAL test**

**Figure 24.** Plot of the $G_1$ versus $G_{II}$ strain energies for $\lambda = 0.1$ and $\lambda = 0.75$. 

**non-conventional techniques** [mixed-mode I+II]
non-conventional techniques [mixed-mode I+II]
it can be concluded that both methods provide consistent results agreement increases as the conditions of self-similar crack propagation (constant FPZ length – Figure 20) become more evident.

Figure 25. Plot of the $G_T$ strain energy for $\lambda = 0.7$ obtained with CCM and CBBM.

validation
Figure 26. Schematic representation of loading scheme 2 with $\lambda = 0.75$.

Figure 27. Envelope for Araldite 2015 with 0.2 mm bondline and $\lambda = 0.75$. 

non-conventional techniques [mixed-mode I+II]
non-conventional techniques \([\text{mixed-mode I+II}]\)

Figure 26. Schematic representation of loading scheme 2 with \(\lambda = 0.75\).

Figure 27. Envelope for Araldite 2015 with 0.2 mm bondline and \(\lambda = 0.75\).

From table 3.

\[
\begin{align*}
G_I &= 0.44 \pm 0.05 \text{ [N/mm]} \\
G_{II} &= 2.1 \pm 0.21 \text{ [N/mm]}
\end{align*}
\]
Mixed-mode testing is being implemented with a specimen load jig similar to the one that Spelt proposed, using DCB specimens used for the pure mode I (DCB) and pure mode II (ENF) and also for mixed-mode DAL.

Figure 28. Load jig specimen geometry.

Figure 29. Specimen tested with the Spelt load jig.
Figures 30, 31, and 32 show P-δ, P-Δa, and R curves for Ψ = 56°, respectively. These figures demonstrate the non-conventional techniques for mixed-mode I+II.
Spelt load jig (ongoing work)

Figure 33. Preliminary envelope for $\Psi = 56^\circ$
The specimen was modelled with 3992 plane strain 8-node quadrilateral elements and 382 6-node interface elements with null thickness placed at the mid-plane of the bonded specimen.

The goal is to develop a CBBM for the Spelt Specimen load jig.

Obtain the envelope for different bondline thicknesses, with this jig and the analytical tool (CBBM).

non-conventional techniques [mixed-mode I+II]
a new data reduction scheme based on specimen compliance, beam theory and crack equivalent concept was proposed to overcome some problems intrinsic to the DAL test

the model provides a simple mode partitioning method and does not require crack length monitoring during the test, which can lead to incorrect estimation of fracture energy due to measurements errors

since the current compliance is used to estimate the equivalent crack length, the method is able to account indirectly for the presence of a non-negligible fracture process zone (very important for ductile adhesives)

for pure modes I and II, excellent agreement was achieved with the fracture values inputted in the cohesive model

in some cases the mixed-mode ratio alters significantly during crack propagation, thus leading to non self-similar crack growth

the positive values of the displacement ratio ($\lambda$) covered almost all the fracture envelope

a slight difference relative to the inputted linear energetic criterion was observed in the central region of the $G_I$ versus $G_{II}$ plot, corresponding to mixed-mode loading, which is attributed to the non self-similar crack propagation conditions that are more pronounced in these cases

only two combinations of the displacement ratio are sufficient to cover almost all the adhesive fracture envelope

conclusions
Develop a CBBM data reduction scheme for the Spelt specimen load jig.

Obtain the envelope for different bondline thicknesses, with this jig and the analytical tool (CBBM).
The authors would like to thank the contribution of Edoardo Nicoli, and Youliang Guan for their work in experimental testing at Virginia Tech.

The authors also acknowledge the financial support of Fundação Luso Americana para o Desenvolvimento (FLAD) through project 314/06, 2007, IDMEC and FEUP.

Thank you.
Thank you.